A Short Semi-Systematic Selection of Indefinite Integrals Classified as Elementary or Nonelementary

#97 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

Infinite Vistas Press
PVD RI
2003

GG97-1 (19)

500 Angell St #414
Providence RI 02906

GG97-2

□ on the basis of personal experience
it is undoubtedly true that
many more indefinite integrals of elementary functions
are nonelementarry than elementary;
altho a quantitative measure may not be evident;
here is a modest list that helps to bolster that judgement;
the examples are suggestive of many more;
these considerations are in the real field ℝ;
x is a real number variable;
NEF stands for
nonelementary function

☐ pattern for seven indefinite integrals based on f(x)

integral	integrand
$\int f(x)\,dx$	fcn
$\int \frac{1}{f(x)}\,dx$	recip of fcn
$\int x\,f(x)\,dx$	x times fcn
$\int \frac{1}{x\,f(x)}\,dx$	recip of x times fcn
$\int \frac{f(x)}{x}\,dx$	fcn divided by x
$\int \frac{x}{f(x)}\,dx$	recip of fcn divided by x
$\int f(f(x))\,dx$	fcn of fcn

☐ $f(x) = e^x$

$$\int e^x dx = e^x + C$$

$$\int \frac{1}{e^x} dx = -e^{-x} + C$$

$$\int x e^x dx = x e^x - e^x + C$$

$$\int \frac{1}{x e^x} dx = \text{NEF}$$

$$\int \frac{e^x}{x} dx = \text{NEF}$$

$$\int \frac{x}{e^x} dx = -x e^{-x} - e^{-x} + C$$

$$\int e^{e^x} dx = \text{NEF}$$

☐ $f(x) = \log x$

$$\int \log x \, dx = x \log x - x + C$$

$$\int \frac{1}{\log x} dx = \text{NEF}$$

$$\int x \log x \, dx = \frac{1}{4} x^2 (2 \log x - 1) + C$$

$$\int \frac{1}{x \log x} dx = \log \log x + C$$

$$\int \frac{\log x}{x} dx = \frac{1}{2} (\log x)^2 + C$$

$$\int \frac{x}{\log x} dx = \text{NEF}$$

$$\int \log \log x \, dx = \text{NEF}$$

☐ $f(x) = \sin x$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \frac{1}{\sin x} dx = \log(\csc x - \cot x) + C = \log \tan \frac{x}{2} + C$$

$$\int x \sin x \, dx = -x \cos x + \sin x + C$$

$$\int \frac{1}{x \sin x} dx = \text{NEF}$$

$$\int \frac{\sin x}{x} dx = \text{NEF}$$

$$\int \frac{x}{\sin x} dx = \text{NEF}$$

$$\int \sin \sin x \, dx = \text{NEF}$$

$\square$ $f(x) = \sin x^2$

$$\int \sin x^2 \, dx = \text{NEF}$$

$$\int \frac{1}{\sin x^2} dx = \text{NEF}$$

$$\int x \sin x^2 \, dx = -\frac{1}{2} \cos x^2 + C$$

$$\int \frac{1}{x \sin x^2} dx = \text{NEF}$$

$$\int \frac{\sin x^2}{x} dx = \text{NEF}$$

$$\int \frac{x}{\sin x^2} dx = \frac{1}{2} \log(\csc x^2 - \cot x^2) + C = \frac{1}{2} \log \tan \frac{x^2}{2} + C$$

$$\int \sin \sin^2 x^2 \, dx = \text{NEF}$$

☐ integrand = trig fcn of log

$$\int \sin \log x \, dx = \frac{1}{2} x (\sin \log x - \cos \log x) + C$$

$$\int \cos \log x \, dx = \frac{1}{2} x (\sin \log x + \cos \log x) + C$$

$$\int \tan \log x \, dx = \text{NEF}$$

$$\int \cot \log x \, dx = \text{NEF}$$

$$\int \sec \log x \, dx = \text{NEF}$$

$$\int \csc \log x \, dx = \text{NEF}$$

☐ integrand = log of trig fcn

$$\int \log \sin x \, dx = \text{NEF}$$

$$\int \log \cos x \, dx = \text{NEF}$$

$$\int \log \tan x \, dx = \text{NEF}$$

$$\int \log \cot x \, dx = \text{NEF}$$

$$\int \log \sec x \, dx = \text{NEF}$$

$$\int \log \csc x \, dx = \text{NEF}$$

☐ integrand $= \sqrt{1+x^n}$ wh $n \in$ pos int

$$\int \sqrt{1+x}\,dx = \frac{2}{3}(1+x)^{\frac{3}{2}} + C$$

$$\int \sqrt{1+x^2}\,dx = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\sinh^{-1}x + C$$

$$\int \sqrt{1+x^n}\,dx = \text{NEF for } n \geq 3$$

$\square$ integrand $= \dfrac{1}{\sqrt{1+x^n}}$ wh $n \in$ pos int

$$\int \frac{1}{\sqrt{1+x}}\,dx = 2\sqrt{1+x} + C$$

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{1+x^n}}\,dx = \text{NEF for } n \geq 3$$

☐ integrand $= \sqrt{1-x^n}$ wh $n \in$ pos int

$$\int \sqrt{1-x}\,dx = -\frac{2}{3}(1-x)^{\frac{3}{2}} + C$$

$$\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x + C$$

$$\int \sqrt{1-x^n}\,dx = \text{NEF for } n \geq 3$$

☐ integrand $= \dfrac{1}{\sqrt{1-x^n}}$ wh $n \in$ pos int

$$\int \frac{1}{\sqrt{1-x}}\,dx = -2\sqrt{1-x} + C$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$

$$\int \frac{1}{\sqrt{1-x^n}}\,dx = \text{NEF for } n \geq 3$$

☐ integrand = expo combo

$$\int e^x \log x \, dx = \text{NEF}$$

$$\int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x) + C$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) + C$$

$$\int e^{x^2} dx = \text{NEF}$$

$$\int e^{-x^2} dx = \text{NEF}$$

$$\int x^x dx = \text{NEF} \qquad \left(\text{note } x^x = e^{x \log x}\right)$$

$$\int x^{-x} dx = \text{NEF} \qquad \left(\text{note } x^{-x} = e^{-x \log x}\right)$$

☐ integrand = sq rt of trig fcn

$$\int \sqrt{\sin x}\, dx = \text{NEF}$$

$$\int \sqrt{\cos x}\, dx = \text{NEF}$$

$$\int \sqrt{\sec x}\, dx = \text{NEF}$$

$$\int \sqrt{\csc x}\, dx = \text{NEF}$$

$$\int \sqrt{\tan x}\, dx$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2\tan x} - 1\right)$$

$$+ \frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2\tan x} + 1\right)$$

$$+ \frac{1}{2\sqrt{2}} \log\left(\sqrt{2\tan x} - 1 - \tan x\right)$$

$$- \frac{1}{2\sqrt{2}} \log\left(\sqrt{2\tan x} + 1 + \tan x\right)$$

$$+ C$$

$$\int \sqrt{\cot x}\, dx$$

$$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2\cot x} - 1\right)$$

$$- \frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2\cot x} + 1\right)$$

$$- \frac{1}{2\sqrt{2}} \log\left(\sqrt{2\cot x} - 1 - \cot x\right)$$

$$+ \frac{1}{2\sqrt{2}} \log\left(\sqrt{2\cot x} + 1 + \cot x\right)$$

$$+ C$$

☐ all NEFs

$$\int \sqrt{2+\sin^2 x}\, dx$$

$$\int \frac{1}{\sqrt{2+\sin^2 x}}\, dx$$

$$\int \sqrt{2-\sin^2 x}\, dx$$

$$\int \frac{1}{\sqrt{2-\sin^2 x}}\, dx$$