Orientation

#37 of Gottschalk's Gestalts

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of the Organization & Exposition
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by Walter Gottschalk

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GG37-2

☐ orientation

D. files

let

• $n \in$ pos int

then

• an n - file

$=_{df}$ an ordered n - tuple of n distinct objects

D. point files

let

• $X \in$ set

• $n \in$ pos int

then

• a point n - file of X

$=_{df}$ an ordered n - tuple

of n distinct points = elements of X

D. arrow files

let

• $m, n \in$ pos int

then

• an arrow m - file of $\mathbb{R}^n$

$=_{df}$ an ordered m - tuple

of m distinct arrows of $\mathbb{R}^n$

with a common initial po int

wh

an arrow $=_{df}$ a plural directed line segment

D. determinants of files

let

• $n \in$ pos int

• $\sigma = \left(P_0, P_1, \cdots, P_n \right) \in$ point $(n+1)$ - file of $\mathbb{R}^n$

then

• the bordered determinant of σ

$=_{df}$ the $(n+1)$st order determinant

whose rows are formed by taking consecutively

the coordinates of the points of σ

prefixed by the number 1

• the arrow determinant of σ

$=_{df}$ the nth order determinant

whose rows are formed by taking consecutively

the coordinates of the points $P_1, P_2, \cdots, P_n$

minus the coordinates of P_0

• the values of the above two determinants are equal

D. oriented files

let

• $n \in$ pos int

• $\sigma = (P_0, P_1, \cdots, P_n) \in$ point $(n+1)$ - file of $\mathbb{R}^n$

• $\tau = \left(\overrightarrow{P_0P_1}, \overrightarrow{P_0P_2}, \cdots, \overrightarrow{P_0P_n}\right) =$ the arrow n - file of $\mathbb{R}^n$

induced by σ

then

• σ

$=_{df}$ positively oriented

or

negatively oriented

or

unoriented

according as the bordered determinant of σ is

positive

or

negative

or

zero

• τ

$=_{df}$ positively oriented

or

negatively oriented

or

unoriented

according as the arrow determinant of σ is

positive

or

negative

or

zero

• σ and τ

are compatibly oriented

D. signed orientations

let

• $n \in$ pos int

then

• the positive / negative orientation of $\mathbb{R}^n$

$=_{df}$ the set of all

positively / negatively oriented point $(n+1)$ - files of $\mathbb{R}^n$

or what is essentially equivalent

the set of all

positively / negatively oriented arrow n - files of $\mathbb{R}^n$

R. notions of orientation for euclidean spaces

• since any euclidean space of whatever dimension,
such as line, plane, 3-space, etc,
can be equipped with a rectangular coordinate system
and thus be identified with $\mathbb{R}^n$ for some positive integer n,
these spaces have the notions of orientation
automatically defined for them