A Semi-Systematic Sampling of Trig Identities: Part I

#23 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms of the Organization & Exposition of Mathematics
by Walter Gottschalk

Infinite Vistas Press
PVD RI
2001

GG23-1 (42)

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Δ fcn A ito sin A

- $\sin A = \sin A$

- $\cos A = \sqrt{1-\sin^2 A}$

- $\tan A = \dfrac{\sin A}{\sqrt{1-\sin^2 A}}$

- $\cot A = \dfrac{\sqrt{1-\sin^2 A}}{\sin A}$

- $\sec A = \dfrac{1}{\sqrt{1-\sin^2 A}}$

- $\csc A = \dfrac{1}{\sin A}$

Δ fcn A ito cos A

- $\sin A = \sqrt{1 - \cos^2 A}$
- $\cos A = \cos A$
- $\tan A = \dfrac{\sqrt{1 - \cos^2 A}}{\cos A}$
- $\cot A = \dfrac{\cos A}{\sqrt{1 - \cos^2 A}}$
- $\sec A = \dfrac{1}{\cos A}$
- $\csc A = \dfrac{1}{\sqrt{1 - \cos^2 A}}$

Δ fcn A ito tan A

- $\sin A = \dfrac{\tan A}{\sqrt{1+\tan^2 A}}$

- $\cos A = \dfrac{1}{\sqrt{1+\tan^2 A}}$

- $\tan A = \tan A$

- $\cot A = \dfrac{1}{\tan A}$

- $\sec A = \sqrt{1+\tan^2 A}$

- $\csc A = \dfrac{\sqrt{1+\tan^2 A}}{\tan A}$

Δ fcn A ito cot A

- $\sin A = \dfrac{1}{\sqrt{1+\cot^2 A}}$

- $\cos A = \dfrac{\cot A}{\sqrt{1+\cot^2 A}}$

- $\tan A = \dfrac{1}{\cot A}$

- $\cot A = \cot A$

- $\sec A = \dfrac{\sqrt{1+\cot^2 A}}{\cot A}$

- $\csc A = \sqrt{1+\cot^2 A}$

Δ fcn A ito sec A

- $\sin A = \dfrac{\sqrt{\sec^2 A - 1}}{\sec A}$

- $\cos A = \dfrac{1}{\sec A}$

- $\tan A = \sqrt{\sec^2 A - 1}$

- $\cot A = \dfrac{1}{\sqrt{\sec^2 A - 1}}$

- $\sec A = \sec A$

- $\csc A = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$

Δ fcn A ito csc A

- $\sin A = \dfrac{1}{\csc A}$

- $\cos A = \dfrac{\sqrt{\csc^2 A - 1}}{\csc A}$

- $\tan A = \dfrac{1}{\sqrt{\csc^2 A - 1}}$

- $\cot A = \sqrt{\csc^2 A - 1}$

- $\sec A = \dfrac{\csc A}{\sqrt{\csc^2 A - 1}}$

- $\csc A = \csc A$

Δ fcn 2A ito sin A

- $\sin 2A = 2\sin A\sqrt{1-\sin^2 A}$
- $\cos 2A = 1-2\sin^2 A$
- $\tan 2A = \dfrac{2\sin A\sqrt{1-\sin^2 A}}{1-2\sin^2 A}$
- $\cot 2A = \dfrac{1-2\sin^2 A}{2\sin A\sqrt{1-\sin^2 A}}$
- $\sec 2A = \dfrac{1}{1-2\sin^2 A}$
- $\csc 2A = \dfrac{1}{2\sin A\sqrt{1-\sin^2 A}}$

Δ fcn 2A ito cos A

- $\sin 2A = 2\cos A\sqrt{1-\cos^2 A}$
- $\cos 2A = 2\cos^2 A - 1$
- $\tan 2A = \dfrac{2\cos A\sqrt{1-\cos^2 A}}{2\cos^2 A - 1}$
- $\cot 2A = \dfrac{2\cos^2 A - 1}{2\cos A\sqrt{1-\cos^2 A}}$
- $\sec 2A = \dfrac{1}{2\cos^2 A - 1}$
- $\csc 2A = \dfrac{1}{2\cos A\sqrt{1-\cos^2 A}}$

Δ fcn 2A ito tan A

- $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$

- $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

- $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

- $\cot 2A = \dfrac{1-\tan^2 A}{2\tan A}$

- $\sec 2A = \dfrac{1+\tan^2 A}{1-\tan^2 A}$

- $\csc 2A = \dfrac{1+\tan^2 A}{2\tan A}$

Δ fcn 2A ito cot A

- $\sin 2A = \dfrac{2\cot A}{\cot^2 A + 1}$

- $\cos 2A = \dfrac{\cot^2 A - 1}{\cot^2 A + 1}$

- $\tan 2A = \dfrac{2\cot A}{\cot^2 A - 1}$

- $\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}$

- $\sec 2A = \dfrac{\cot^2 A + 1}{\cot^2 A - 1}$

- $\csc 2A = \dfrac{\cot^2 A + 1}{2\cot A}$

Δ fcn 2A ito sec A

- $\sin 2A = \dfrac{2\sqrt{\sec^2 A - 1}}{\sec^2 A}$

- $\cos 2A = \dfrac{2 - \sec^2 A}{\sec^2 A}$

- $\tan 2A = \dfrac{2\sqrt{\sec^2 A - 1}}{2 - \sec^2 A}$

- $\cot 2A = \dfrac{2 - \sec^2 A}{2\sqrt{\sec^2 A - 1}}$

- $\sec 2A = \dfrac{\sec^2 A}{2 - \sec^2 A}$

- $\csc 2A = \dfrac{\sec^2 A}{2\sqrt{\sec^2 A - 1}}$

Δ fcn 2A ito csc A

- $\sin 2A = \dfrac{2\sqrt{\csc^2 A - 1}}{\csc^2 A}$

- $\cos 2A = \dfrac{\csc^2 A - 2}{\csc^2 A}$

- $\tan 2A = \dfrac{2\sqrt{\csc^2 A - 1}}{\csc^2 A - 2}$

- $\cot 2A = \dfrac{\csc^2 A - 2}{2\sqrt{\csc^2 A - 1}}$

- $\sec 2A = \dfrac{\csc^2 A}{\csc^2 A - 2}$

- $\csc 2A = \dfrac{\csc^2 A}{2\sqrt{\csc^2 A - 1}}$

Δ fcn 2A ito sin A & cos A

- $\sin 2A = 2 \sin A \cos A$

- $\cos 2A = \cos^2 A - \sin^2 A$

- $\tan 2A = \dfrac{2 \sin A \cos A}{\cos^2 A - \sin^2 A}$

- $\cot 2A = \dfrac{\cos^2 A - \sin^2 A}{2 \sin A \cos A}$

- $\sec 2A = \dfrac{1}{\cos^2 A - \sin^2 A}$

- $\csc 2A = \dfrac{1}{2 \sin A \cos A}$

Δ fcn 2A ito $\cot A \pm \tan A$

- $\sin 2A = \dfrac{2}{\cot A + \tan A}$

- $\cos 2A = \dfrac{\cot A - \tan A}{\cot A + \tan A}$

- $\tan 2A = \dfrac{2}{\cot A - \tan A}$

- $\cot 2A = \dfrac{\cot A - \tan A}{2}$

- $\sec 2A = \dfrac{\cot A + \tan A}{\cot A - \tan A}$

- $\csc 2A = \dfrac{\cot A + \tan A}{2}$

Δ fcn 2A ito sec A & csc A

- $\sin 2A = \dfrac{2}{\csc A \sec A}$

- $\cos 2A = \dfrac{\csc^2 A - \sec^2 A}{\csc^2 A + \sec^2 A} = \dfrac{\csc^2 A - \sec^2 A}{\csc^2 A \sec^2 A}$

- $\tan 2A = \dfrac{2 \csc A \sec A}{\csc^2 A - \sec^2 A}$

- $\cot 2A = \dfrac{\csc^2 A - \sec^2 A}{2 \csc A \sec A}$

- $\sec 2A = \dfrac{\csc^2 A + \sec^2 A}{\csc^2 A - \sec^2 A} = \dfrac{\csc^2 A \sec^2 A}{\csc^2 A - \sec^2 A}$

- $\csc 2A = \dfrac{\csc A \sec A}{2}$

Δ fcn 2A ito fcn A

- $\sin 2A = 2\sin A\sqrt{1-\sin^2 A}$

- $\cos 2A = 2\cos^2 A - 1$

- $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

- $\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}$

- $\sec 2A = \dfrac{\sec^2 A}{2-\sec^2 A}$

- $\csc 2A = \dfrac{\csc^2 A}{2\sqrt{\csc^2 A - 1}}$

Δ fcn 3A ito fcn A

- $\sin 3A = 3\sin A - 4\sin^3 A$

- $\cos 3A = 4\cos^3 A - 3\cos A$

- $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

- $\cot 3A = \dfrac{3\cot A - \cot^3 A}{1 - 3\cot^2 A}$

- $\sec 3A = \dfrac{\sec^3 A}{4 - 3\sec^2 A}$

- $\csc 3A = \dfrac{\csc^3 A}{3\csc^2 A - 4}$

Δ fcn4A ito fcnA

- $\sin 4A = 4\sin A\sqrt{1-\sin^2 A} - 8\sin^3 A\sqrt{1-\sin^2 A}$

- $\cos 4A = 8\cos^4 A - 8\cos^2 A + 1$

- $\tan 4A = \dfrac{4\tan A - 4\tan^3 A}{1 - 6\tan^2 A + \tan^4 A}$

- $\cot 4A = -\dfrac{1 - 6\cot^2 A + \cot^4 A}{4\cot A - 4\cot^3 A}$

- $\sec 4A = \dfrac{\sec^4 A}{8 - 8\sec^2 A + \sec^4 A}$

- $\csc 4A = \dfrac{\csc^4 A}{4\csc^2 A\sqrt{\csc^2 - 1} - 8\sqrt{\csc^2 A - 1}}$

Δ fcn5A ito fcnA

- $\sin 5A = 5\sin A - 20\sin^3 A + 16\sin^5 A$

- $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

- $\tan 5A = \dfrac{5\tan A - 10\tan^3 A + \tan^5 A}{1 - 10\tan^2 A + 5\tan^4 A}$

- $\cot 5A = \dfrac{5\cot A - 10\cot^3 A + \cot^5 A}{1 - 10\cot^2 A + 5\cot^4 A}$

- $\sec 5A = \dfrac{\sec^5 A}{16 - 20\sec^2 A + 5\sec^4 A}$

- $\csc 5A = \dfrac{\csc^5 A}{5\csc^4 A - 20\csc^2 A + 16}$

Δ $\sin nA$ ito $\sin A$ & $\cos A$ ($n \in \underline{7}$)

- $\sin A = \sin A$
- $\sin 2A = 2\sin A\cos A$
- $\sin 3A = 3\sin A - 4\sin^3 A$
- $\sin 4A = 4\sin A\cos A - 8\sin^3 A\cos A$
- $\sin 5A = 5\sin A - 20\sin^3 A + 16\sin^5 A$
- $\sin 6A = 6\sin A\cos A - 32\sin^3 A\cos A + 32\sin^5 A\cos A$
- $\sin 7A = 7\sin A - 56\sin^3 A + 112\sin^5 A - 64\sin^7 A$

Δ $\cos nA$ ito $\cos A$ $(n \in \underline{7})$

- $\cos A = \cos A$
- $\cos 2A = 2\cos^2 A - 1$
- $\cos 3A = 4\cos^3 A - 3\cos A$
- $\cos 4A = 8\cos^4 A - 8\cos^2 A + 1$
- $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$
- $\cos 6A = 32\cos^6 A - 48\cos^4 A + 18\cos^2 A - 1$
- $\cos 7A = 64\cos^7 A - 112\cos^5 A + 56\cos^3 A - 7\cos A$

Δ De Moivre's theorem

- $(\cos A + i \sin A)^n = \cos nA + i \sin nA \quad (n \in \text{pos int})$

which gives
the general multiple - angle formulas
for cos & sin
when the LHS is expanded
by the binomial theorem
& real parts are equated
& imaginary parts are equated
viz

- $\cos nA$

$$= \binom{n}{0}\cos^n A - \binom{n}{2}\cos^{n-2} A \sin^2 A$$

$$+\binom{n}{4}\cos^{n-4} A \sin^4 A - \binom{n}{6}\cos^{n-6} A \sin^6 A + \cdots$$

$$= \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} (-1)^k \binom{n}{2k} \cos^{n-2k} A \sin^{2k} A \qquad \text{wh } n \in \text{pos int}$$

- $\sin nA$

$$= \binom{n}{1}\cos^{n-1} A \sin A - \binom{n}{3}\cos^{n-3} A \sin^3 A$$

$$+\binom{n}{5}\cos^{n-5} A \sin^5 A - \binom{n}{7}\cos^{n-7} A \sin^7 A + \cdots$$

$$= \sum_{k=1}^{\left\lceil \frac{n}{2} \right\rceil} (-1)^{k-1} \binom{n}{2k-1} \cos^{n-2k+1} A \sin^{2k-1} A \qquad \text{wh } n \in \text{pos int}$$

Δ other forms
of the general multiple - angle formulas
for sin & cos

- $\cos nA$

$$= 2^{n-1}\binom{n}{0}\cos^n A - \frac{n}{n-1}2^{n-3}\binom{n-1}{1}\cos^{n-2} A$$

$$+\frac{n}{n-2}2^{n-5}\binom{n-2}{2}\cos^{n-4} A - \frac{n}{n-3}2^{n-7}\binom{n-3}{3}\cos^{n-6} A + \cdots$$

$$= \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor}(-1)^k \frac{n}{n-k}2^{n-2k-1}\binom{n-k}{k}\cos^{n-2k} A$$

wh $n \in$ pos int

- $\cos nA$

$$= (-1)^{\frac{n}{2}}\left[\, 2^{n-1}\binom{n}{0}\sin^{n} A - \frac{n}{n-1}2^{n-3}\binom{n-1}{1}\sin^{n-2} A\right.$$

$$\left. + \frac{n}{n-2}2^{n-5}\binom{n-2}{2}\sin^{n-4} A - \frac{n}{n-3}2^{n-7}\binom{n-3}{3}\sin^{n-6} A + \cdots\right]$$

$$= (-1)^{\frac{n}{2}}\sum_{k=0}^{\frac{n}{2}}(-1)^{k}\frac{n}{n-k}2^{n-2k-1}\binom{n-k}{k}\sin^{n-2k} A$$

wh $n \in$ even pos int

- $\cos nA$

$$= (-1)^{\frac{n-1}{2}}\cos A\left[\, 2^{n-1}\binom{n-1}{0}\sin^{n-1} A - 2^{n-3}\binom{n-2}{1}\sin^{n-3} A\right.$$

$$\left. + 2^{n-5}\binom{n-3}{2}\sin^{n-5} A - 2^{n-7}\binom{n-4}{3}\sin^{n-7} A + \cdots\right]$$

$$= (-1)^{\frac{n-1}{2}}\cos A\sum_{k=0}^{\frac{n-1}{2}}(-1)^{k}2^{n-2k-1}\binom{n-k-1}{k}\sin^{n-2k-1} A$$

wh $n \in$ odd pos int

- $\sin nA$

$$= \sin A\,[\,2^{n-1}\binom{n-1}{0}\cos^{n-1}A - 2^{n-3}\binom{n-2}{1}\cos^{n-3}A$$

$$+2^{n-5}\binom{n-3}{2}\cos^{n-5}A - 2^{n-3}\binom{n-4}{3}\cos^{n-7}A + \cdots]$$

$$= \sin A \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} (-1)^k\, 2^{n-2k-1}\binom{n-k-1}{k}\cos^{n-2k-1}A$$

wh $n \in$ pos int

- $\sin nA$

$$= (-1)^{\frac{n}{2}-1}\cos A\Big[\,2^{n-1}\binom{n-1}{0}\sin^{n-1}A - 2^{n-3}\binom{n-2}{1}\sin^{n-3}A$$

$$+2^{n-5}\binom{n-3}{2}\sin^{n-5}A - 2^{n-7}\binom{n-4}{3}\sin^{n-7}A + \cdots]$$

$$= (-1)^{\frac{n}{2}-1}\cos A\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}(-1)^k\,2^{n-2k-1}\binom{n-k-1}{k}\sin^{n-2k-1}A$$

wh $n \in$ even pos int

- $\sin nA$

$$= (-1)^{\frac{n-1}{2}}\Big[\,2^{n-1}\binom{n}{0}\sin^{n}A - \frac{n}{n-1}2^{n-3}\binom{n-1}{1}\sin^{n-2}A$$

$$+\frac{n}{n-2}2^{n-5}\binom{n-2}{2}\sin^{n-4}A - \frac{n}{n-3}2^{n-7}\binom{n-3}{3}\sin^{n-6}A + \cdots]$$

$$= (-1)^{\frac{n-1}{2}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}(-1)^k\frac{n}{n-k}2^{n-2k-1}\binom{n-k}{k}\sin^{n-2k}A$$

wh $n \in$ odd pos int

Δ fcn A ito $\tan\frac{A}{2} = t$

- $\sin A = \frac{2t}{1+t^2}$

- $\cos A = \frac{1-t^2}{1+t^2} = \frac{2}{1+t^2} - 1$

- $\tan A = \frac{2t}{1-t^2} = \frac{1}{1-t} - \frac{1}{1+t}$

- $\cot A = \frac{1-t^2}{2t} = \frac{1}{2t} - \frac{t}{2}$

- $\sec A = \frac{1+t^2}{1-t^2} = \frac{1}{1-t} + \frac{1}{1+t} - 1$

- $\csc A = \frac{1+t^2}{2t} = \frac{1}{2t} + \frac{t}{2}$

which are all simple rational functions of t

Δ also note for $\tan\dfrac{A}{2} = t$

- $\sec A + \tan A = \dfrac{1+t}{1-t} = \dfrac{2}{1-t} - 1$

- $\sec A - \tan A = \dfrac{1-t}{1+t} = \dfrac{2}{1+t} - 1$

- $\csc A + \cot A = \cot\dfrac{A}{2} = \dfrac{1}{t}$

- $\csc A - \cot A = \tan\dfrac{A}{2} = t$

Δ fcn A ito $\cot\dfrac{A}{2} = t$

- $\sin A = \dfrac{2t}{1+t^2}$

- $\cos A = \dfrac{t^2-1}{t^2+1} = 1 - \dfrac{2}{t^2+1}$

- $\tan A = \dfrac{2t}{t^2-1} = \dfrac{1}{t+1} + \dfrac{1}{t-1}$

- $\cot A = \dfrac{t^2-1}{2t} = \dfrac{t}{2} - \dfrac{1}{2t}$

- $\sec A = \dfrac{t^2+1}{t^2-1} = 1 + \dfrac{1}{t-1} - \dfrac{1}{t+1}$

- $\csc A = \dfrac{t^2+1}{2t} = \dfrac{t}{2} + \dfrac{1}{2t}$

which are all simple rational functions of t

Δ also note for $\cot\frac{A}{2} = t$

- $\sec A + \tan A = \frac{t+1}{t-1} = 1 + \frac{2}{t-1}$

- $\sec A - \tan A = \frac{t-1}{t+1} = 1 - \frac{2}{t+1}$

- $\csc A + \cot A = \cot\frac{A}{2} = t$

- $\csc A - \cot A = \tan\frac{A}{2} = \frac{1}{t}$

$$\Delta \ \text{fcn} \frac{A}{2} \ \text{ito} \ \sin A$$

- $$\sin\frac{A}{2} = \sqrt{\frac{1-\sqrt{1-\sin^2 A}}{2}}$$

- $$\cos\frac{A}{2} = \sqrt{\frac{1+\sqrt{1-\sin^2 A}}{2}}$$

- $$\tan\frac{A}{2} = \frac{1-\sqrt{1-\sin^2 A}}{\sin A}$$

- $$\cot\frac{A}{2} = \frac{1+\sqrt{1-\sin^2 A}}{\sin A}$$

- $$\sec\frac{A}{2} = \frac{\sqrt{2\left(1-\sqrt{1-\sin^2 A}\right)}}{\sin A}$$

- $$\csc\frac{A}{2} = \frac{\sqrt{2\left(1+\sqrt{1-\sin^2 A}\right)}}{\sin A}$$

$$\Delta \ \text{fcn}\,\frac{A}{2} \ \ \text{ito} \ \cos A$$

- $\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$

- $\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$

- $\tan\frac{A}{2} = \sqrt{\frac{1-\cos A}{1+\cos A}}$

- $\cot\frac{A}{2} = \sqrt{\frac{1+\cos A}{1-\cos A}}$

- $\sec\frac{A}{2} = \sqrt{\frac{2}{1+\cos A}}$

- $\csc\frac{A}{2} = \sqrt{\frac{2}{1-\cos A}}$

$$\Delta \ \text{fcn}\,\frac{A}{2} \ \text{ito} \ \tan A$$

- $\sin\frac{A}{2} = \sqrt{\frac{\sqrt{1+\tan^2 A}-1}{2\sqrt{1+\tan^2 A}}}$

- $\cos\frac{A}{2} = \sqrt{\frac{\sqrt{1+\tan^2 A}+1}{2\sqrt{1+\tan^2 A}}}$

- $\tan\frac{A}{2} = \frac{\sqrt{1+\tan^2 A}-1}{\tan A}$

- $\cot\frac{A}{2} = \frac{\sqrt{1+\tan^2 A}+1}{\tan A}$

- $\sec\frac{A}{2} = \frac{\sqrt{2\left(1+\tan^2 A-\sqrt{1+\tan^2 A}\right)}}{\tan A}$

- $\csc\frac{A}{2} = \frac{\sqrt{2\left(1+\tan^2 A+\sqrt{1+\tan^2 A}\right)}}{\tan A}$

$$\Delta \ \text{fcn}\,\frac{A}{2} \ \ \text{ito} \ \cot A$$

- $$\sin\frac{A}{2} \ = \ \sqrt{\frac{\sqrt{1+\cot^2 A}-\cot A}{2\sqrt{1+\cot^2 A}}}$$

- $$\cos\frac{A}{2} \ = \ \sqrt{\frac{\sqrt{1+\cot^2 A}+\cot A}{2\sqrt{1+\cot^2 A}}}$$

- $$\tan\frac{A}{2} \ = \ \sqrt{1+\cot^2 A}-\cot A$$

- $$\cot\frac{A}{2} \ = \ \sqrt{1+\cot^2 A}+\cot A$$

- $$\sec\frac{A}{2} \ = \ \sqrt{2\left(1+\cot^2 A-\cot A\sqrt{1+\cot^2 A}\right)}$$

- $$\csc\frac{A}{2} \ = \ \sqrt{2\left(1+\cot^2 A+\cot A\sqrt{1+\cot^2 A}\right)}$$

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Δ fcn $\frac{A}{2}$ ito sec A

- $\sin\frac{A}{2} = \sqrt{\frac{\sec A - 1}{2\sec A}}$

- $\cos\frac{A}{2} = \sqrt{\frac{\sec A + 1}{2\sec A}}$

- $\tan\frac{A}{2} = \sqrt{\frac{\sec A - 1}{\sec A + 1}}$

- $\cot\frac{A}{2} = \sqrt{\frac{\sec A + 1}{\sec A - 1}}$

- $\sec\frac{A}{2} = \sqrt{\frac{2\sec A}{\sec A + 1}}$

- $\csc\frac{A}{2} = \sqrt{\frac{2\sec A}{\sec A - 1}}$

Δ fcn $\frac{A}{2}$ ito csc A

- $\sin\frac{A}{2} = \sqrt{\frac{\csc A - \sqrt{\csc^2 A - 1}}{2\csc A}}$

- $\cos\frac{A}{2} = \sqrt{\frac{\csc A + \sqrt{\csc^2 A - 1}}{2\csc A}}$

- $\tan\frac{A}{2} = \csc A - \sqrt{\csc^2 A - 1}$

- $\cot\frac{A}{2} = \csc A + \sqrt{\csc^2 A - 1}$

- $\sec\frac{A}{2} = \sqrt{2\csc A\left(\csc A - \sqrt{\csc^2 A - 1}\right)}$

- $\csc\frac{A}{2} = \sqrt{2\csc A\left(\csc A + \sqrt{\csc^2 A - 1}\right)}$

Δ fcn $\frac{A}{2}$ ito sin A & cos A

- $\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$

- $\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$

- $\tan\frac{A}{2} = \sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$

- $\cot\frac{A}{2} = \sqrt{\frac{1+\cos A}{1-\cos A}} = \frac{1+\cos A}{\sin A} = \frac{\sin A}{1-\cos A}$

- $\sec\frac{A}{2} = \sqrt{\frac{2}{1+\cos A}} = \frac{\sqrt{2(1-\cos A)}}{\sin A}$

- $\csc\frac{A}{2} = \sqrt{\frac{2}{1-\cos A}} = \frac{\sqrt{2(1+\cos A)}}{\sin A}$

$$\Delta \ \text{fcn}\,\frac{A}{2} \quad \text{ito} \quad \tan A \ \& \ \sec A$$

- $\sin\frac{A}{2} = \sqrt{\frac{\sec A - 1}{2\sec A}}$

- $\cos\frac{A}{2} = \sqrt{\frac{\sec A + 1}{2\sec A}}$

- $\tan\frac{A}{2} = \sqrt{\frac{\sec A - 1}{\sec A + 1}} = \frac{\sec A - 1}{\tan A} = \frac{\tan A}{\sec A + 1}$

- $\cot\frac{A}{2} = \sqrt{\frac{\sec A + 1}{\sec A - 1}} = \frac{\sec A + 1}{\tan A} = \frac{\tan A}{\sec A - 1}$

- $\sec\frac{A}{2} = \sqrt{\frac{2\sec A}{\sec A + 1}} = \frac{\sqrt{2\sec A(\sec A - 1)}}{\tan A}$

- $\csc\frac{A}{2} = \sqrt{\frac{2\sec A}{\sec A - 1}} = \frac{\sqrt{2\sec A(\sec A + 1)}}{\tan A}$

$$\Delta \ \text{fcn}\,\frac{A}{2} \quad \text{ito} \quad \cot A \,\&\, \csc A$$

- $$\sin\frac{A}{2} = \sqrt{\frac{\csc A - \cot A}{2\csc A}}$$

- $$\cos\frac{A}{2} = \sqrt{\frac{\csc A + \cot A}{2\csc A}}$$

- $$\tan\frac{A}{2} = \csc A - \cot A$$

- $$\cot\frac{A}{2} = \csc A + \cot A$$

- $$\sec\frac{A}{2} = \sqrt{2\csc A(\csc A - \cot A)}$$

- $$\csc\frac{A}{2} = \sqrt{2\csc A(\csc A + \cot A)}$$