Math Snippets: Third Bouquet

#30 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

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☐ Euler's constant

• definition of Euler's constant γ

$\gamma =_{df} \lim_{n\to\infty} (H_n - \log n)$ we

wh

n $\in$ pos int var

&

H_n

= the nth harmonic number

= the nth partial sum of the harmonic series

$= 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$

note that the existence of Euler's constant says that
the harmonic series behaves like the logarithm function
& its value tells the discrepancy

the value of Euler's constant to ten decimal places is
γ = 0.57721 56649 +

• integral expressions for Euler' s constant γ

$$\gamma = -\int_0^1 \log|\log x|\,dx$$

$$\gamma = -\int_0^\infty \frac{\log x}{e^x}\,dx$$

• Euler' s constant γ ito of the zeta function

$$\gamma = \lim_{x \to 1+} \left[\zeta(x) - \frac{1}{x-1}\right]$$

wh

$x \in$ real var

$\zeta(z)$ is a meromorphic function

whose only pole is

a simple pole with residue 1 at $z = 1$;

$\zeta(z) - \frac{1}{z-1}$ is an entire function

whose value at $z = 1$ is γ

☐ Euler's
excellent & famous
elegant & fabulous
enchanting & fantastic
exciting & fascinating
prime-producing polynomial
=
Euler's forty-one formula
is
the monic quadratic polynomial
with positive integer coefficients

- $n^2 + n + 41 \quad (n \in \text{int var})$

which gives
40 distinct primes
for the 40 consecutive integer values of n
from n = 0 to n = 39
viz
41, 43, 47, 53, 61,
71, 83, 97, 113, 131,
151, 173, 197, 223, 251,
281, 313, 347, 383, 421,
461, 503, 547, 593, 641,
691, 743, 797, 853, 911,
971, 1033. 1097, 1163, 1231,
1301, 1373, 1447, 1523, 1601
&
which repeats these 40 primes
from n = –1 to n = – 40;

when n = 40 & again when n = – 41,

the value of the polynomial is the square 1681of 41

• the quadratic polynomial

$$36n^2 - 810n + 2753$$

gives 45 prime numbers
for the 45 consecutive integer values of n
from n = 0 to n = 44

• bioline
Leonhard Euler
1707-1783
Swiss, lived many years in Germany & Russia
algebraist, analyst, geometer,
number theorist, probabilist,
applied mathematician, calculating prodigy;
most prolific mathematician of all time

□ mathematics & a touch of mysticism

Δ ouroboros

= oo-ROB-uh-russ
= a dragon/serpent/snake biting/swallowing its own tail
from
ουρα (Greek)
= tail
+
βορος (Greek)
= devouring

Δ the ouroboros

is usually represented
in the shape of a circle ○
but it may occur
in the shape of an infinity sign ∞

Δ the ouroboros

is an ancient world-wide mystical symbol
with many meanings
&
with strangely mathematical overtones
eg
the ouroboros symbolizes

• constant rejuvenation

• the continuity of life

• cyclic time
= die ewige Wiederkunft (German; Nietzsche)
= le retour éternel (French)
= eternal recurrence
= eternal reemergence
= eternal return
= the doctrine that the world is periodic

• descent of spirit into matter & return ascent

• disintegration & reintegration
= ‘my end is my beginning’

• the eternity of time

• immortality

• the infinity of space

• truth & cognition

• wisdom

• in alchemy:
the passage from solid to liquid to gas
& the reverse

• Buddhist/Hindu meaning:
the wheel of samsara

• Egyptian meaning:
the circle of the universe;
the path of the sun

• Greek meaning:
`εν το παν (Greek)
= lit: the one, the all
= all is one

☐ the political distinction
between
a necessary condition
and
a sufficient condition

• a necessary condition nominates
but does not elect

• a sufficient condition elects

☐ ¿ Platonic Ideas/Forms = sets ?

• the Platonic Ideas/Forms
constitute a forerunner of
the notion of set

• eg
in 'a chair is a shadow of the Idea of Chair'
'the Idea of Chair'
could be correlated with
'the set of all chairs';
&
'is a shadow of'
could be correlated with
'is an element of'

• in tabular form

Plato	set theory
a chair	a chair
is a shadow of	is an element of
the Idea of Chair	the set of all chairs

☐ Platonism vs nominalism

• Platonic realism
= Platonism
= realism
= the philosophical thesis that abstract objects,
such as concepts and mathematical objects,
have an independent actual real (whence the name)
existence which is equal to, or even superior to,
the existence of physical objects;
first promulgated by Plato (whence the name);
opposed to nominalism

• nominalism
= the philosophical thesis that abstract objects,
such as concepts and mathematical objects,
do not exist
ie
abstract objects exist in name only
= abstract objects exist only nominally
= abstract objects have only a nominal existence
= the names exist but the objects do not exist
(whence the name ‘nominalism’);
opposed to Platonic realism

☐ an opposing pair of philosophical words

• phenomenon
= fee-NOM-ee-NON
= that which appears to the senses
= our perception of a thing
from
φαινομενον (Greek)
= appearance

• noumenon
= NOO-mee-NON
= the thing behind our perception of it
= das Ding an sich (German; Kant)
= lit: the thing in itself
from
νουμενον (Greek)
= that which is perceived

☐ physiological aspects of mathematics

• the language of mathematics
is much more
graphic = written/printed/seen
than
auditory = spoken/heard

• it is likely that
seeing & hearing
mathematical exposition at the same time
(often together with writing)
helps in the learning/understanding process;
but to use the auditory medium alone
for the communication of mathematics
ie without graphic accompaniment
requires high expertise
on the part of both speaker & hearer

• of the five human physiological senses
seeing
hearing
touching
smelling
tasting
it seems that only the first three senses
seeing
hearing
touching
are involved in
the recognition & expression & communication
of mathematics
& in decreasing order of importance;
the last two senses
smelling
tasting
do not seem to be involved at all

• it is likely that
the sense of touch
&
the proprioceptive perception
of bone/muscle movement
are much involved in the formation & recognition
of geometric/spacial notions
&
of algebraic/temporal notions
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• instruments of composition
for the use of individuals engaged in the
discovery/invention/recording/learning/teaching
of mathematics
have been mainly
the traditional paper-and-pencil/ink kinds for two millenia;
but now
the electronic computer/printer has arrived
as a new & powerful composing instrument;
this basicly important change means that
the actual physiology involved in the production of
mathematics will be changing,
presumably for the better

□ the two meanings of ‘orientation’

∆ there are two mathematical/scientific meanings
of the noun ‘orientation’

∆ the first/primary/preferred meaning
of the word ‘orientation’
is related to the words:
• bias
• rotation
• sense
• spin
• turn
• torque
• twirl
• twist
this meaning of ‘orientation’ is
two-valued as eg
• clockwise vs counterclockwise
• direct image vs mirror image
• forward/forwards vs backward/backwards
• positive vs negative
• right/rightward/rightwards vs left/leftward/leftwards
• right-handed vs left-handed
• this way vs that way
• up/upward/upwards vs down/downward/downwards
• yes vs no
etc

∆ the second/secondary/other meaning
of the word ‘orientation’
is to be fully distinguished from the first meaning;
this other meaning is suggested by the words:
• attitude
• bearing
• direction
• pose
• position
• posture
• stance
& generally carries the idea of
• relationship to the environment/surroundings;
this meaning of ‘orientation’
is infinitely multiple-valued eg
the orientation of a rocket ship in space
= the attitude of a rocket ship in space
is described say by the three direction angles
of the axis of the ship
& is specified by the values of the direction angles

∆ to summarize suggestively:
• orientation in the first meaning
is an internal/intrinsic condition
• orientation in the second meaning
is an external/extrinsic condition

Δ etymology

orientation (English)
from
orientation (French noun) = orientation
from
orienter (French verb) = to set toward the east
from
orient (French noun) = the east
from
orientem (acc case)
from
oriens (Latin noun, nom case) = the east, the rising sun
from
oriens (pres part)
from
orior (Latin verb) = to become visible, to rise
from
*or- (Indo-European root) = to raise, to set in motion

☐ mathematics = the study of abstract patterns

the study of

this kind of physical pattern	produces	this kind of mathematics
↓		↓
• chance	.....	probability & statistics
• communication	.....	information theory
• counting	.....	number theory & algebra
• drawing/writing	.....	math notation
• human conflict	.....	game theory
• language	.....	logic
• motion	.....	analysis & applied math

- multiplicity ..... theory of sets & combinatorics
- nearness ..... topology
- perspective ..... projective geometry
- pictures ..... geometry
- space ..... geometry
- symmetry ..... group theory
- time ..... theory of order & analysis
- vibrations/waves ..... theory of Fourier series
- visual/tactile perception ..... geometry

☐ polygonal numbers

are the numbers of dots

arranged in polygonal patterns;

the following algebraic definition

is a consequence

D. polygonal numbers

let

$k, n \in \text{int st } k \geq 3 \ \& \ n \geq 1$

then

the polygonal number of order k & of index n

= the nth k - order polygonal number

$=_{dn} P(k,n)$

$=_{df} \frac{n}{2}[(k-2)n - k + 4]$

wh

polygonal of order 3 = triangular

polygonal of order 4 = square

polygonal of order 5 = pentagonal

polygonal of order 6 = hexagonal

⋮

polygonal of order k = k – gonal

☐ a philosophical/poetical definition
of mathematics & its major branches

mathematics
arises from
the study of

- multiplicity
- space
- time
- motion

where

the study of this phenomenon	generates	this branch of mathematics
↓		↓
• multiplicity	.....	algebra
• space	.....	geometry/topology
• time	.....	analysis
• motion	.....	applied mathematics

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☐ longer periods of time

• second = sec = s
= one adult human heart beat approx

• minute = min = m
= 60 seconds

• hour = hr = h
= 60 minutes

• day = da = d
= 24 hours
from the period of the Earth's rotation

• week = wk = w
= 7 days
(?) from the time-length of
the four interval phases of the Moon ie
waxing crescent, waxing gibbous,
waning gibbous, waning crescent
which is the same thing as
the time-length between
the consecutive phases of the Moon:
new moon, first quarter moon,
full moon, last quarter moon

• fortnight
= 2 weeks
(?) from the time-length between
new moon & full moon

• month = mo
= 28 to 31 days
from the period of the Moon's revolution
about the Earth

• bimester = bim
= 2 months

• trimester = trim
= 3 months

• semester = sem
= 15 to 18 weeks

• academic year
= school year
= the ten months period
from September to June inclusive

• year = yr = y
= 12 months
from the period of the Earth's revolution
about the Sun

• common year = com yr = cy
= 365 days

• leap year = lp yr = ly
= 366 days

- annus

=1 year

- biennium

= 2 years

- triennium

= 3 years

- quadrennium

= 4 years

- quinquennium

= 5 years

- sexennium

= 6 years

- septennium

= 7 years

- octennium

= 8 years

- novennium

= 9 years

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• decade
= decennium
= 10 years

• undecennium
= 11 years

• duodecennium
= 12 years

• tredecennium
= 13 years

• quattuordecennium
= 14 years

• quindecennium
= 15 years

• vicennium
= 20 years

• semicentennium
= 50 years

• century
= centennium
= 100 years

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• sesquicentennium
= 150 years

• bicentennium
= 200 years

• tricentennium
= 300 years

• quadricennium
= 400 years

• quincentennium
= 500 years

• sexcentennium
= 600 years

• septicentennium
= 700 years

• octocentennium
= 800 years

• novecentennium
= 900 years

• millenium
= 1000 years

• myriad
= 10,000 years
(suggested use)

• lac (Hindi)
= 100,000 years
(suggested use)

• geon (blend of geologic + eon)
= 1,000,000 years
(a convenient unit of geologic time)

• crore (Hindi)
= 10,000,000 years
(suggested use)

• era
= 100,000,000 years
(suggested use)

• eon
= 1,000,000,000 years